

Weyl Invariant p -brane and Dp -brane actions

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Abstract

Conformal invariant new forms of p -brane and Dp -brane actions are proposed. The field content of these actions are: an induced metric, gauge fields, an auxiliary metric and an auxiliary scalar field that implements the Weyl invariance. This scalar field transforms with a conformal weight that depends on the brane dimension. The proposed actions are Weyl invariant in any dimension and the elimination of the auxiliary metric and the scalar field reproduces the Nambu-Goto action for p -branes and the Born-Infeld action for Dp -branes. As a consequence of the fact that in the p -brane case, the action is quadratic in $\partial_0 X$, we develop a complete construction for the associated canonical formalism, solving the problem in previous formulations of conformal p -brane actions where the Hamiltonian can not be constructed. We obtain the algebra of constraints and identify the generator of the Weyl symmetry. In the construction of the corresponding supersymmetric generalization of this conformal p -brane action we show that the associated κ -symmetry is consistent with the conformal invariance. In the Dp -brane case, the actions are quadratic in the gauge fields. These actions can be used to construct new conformal couplings in any dimension p to the auxiliary scalar field now promoted as a dynamical variable.

1 Introduction

The underlying extended objects that allowed recent progress in string theory, are the p -branes [1] and Dirichlet branes [2], or Dp -branes. The p -branes are extended structures embedded in a higher dimensional space-time from which it inherits an induced metric. The dynamical properties of such objects are described by the Nambu-Goto action and its generalizations for p -branes that are now proportional to the world-volume. For the case $p = 1$ (string theory) the Nambu-Goto action, given in terms of the area of the string worldsheet can be replaced with a classically equivalent action involving an auxiliary worldsheet metric and local conformal symmetry [3]. In contrast with the non-polynomial Nambu-Goto action the new action is quadratic in the derivatives of the coordinates.

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The introduction of the worldsheet metric as an auxiliary field admits a covariant gauge, simplifying the analysis and allowing a covariant quantization [4]. Conformal invariance or rather Weyl invariance as local worldvolume symmetry can be implemented for all extended objects, not just strings ($p = 1$). A proposal for the construction of a Weyl invariant action for any p [5] gives a higher non-polynomial action that prevents the construction of the associated canonical formalism as well as the study of symmetries and quantum properties. An attempt to develop the canonical analysis of this type of actions was proposed in [6]. Here, we present a different solution to this problem. We construct a new Weyl invariant action using an auxiliary scalar field, for which the standard rules of the canonical analysis can be applied.

On the other hand the Dp -brane is the $(p+1)$ dimensional hypersurface in space-time where the open strings can end and its dynamics is induced by the open strings attached on it. They have been crucial in diverse topics like dualities [7], black hole physics [8], AdS/CFT correspondence [9] and M-theory [10]. The action that describes the dynamics of these objects is of Born-Infeld (BI) type [11]. An action which is quadratic in the gauge fields has been proposed in [12] –based on the introduction of an auxiliary worldvolume metric γ_{ij} – that is conformal invariant only for the case $p = 3$ corresponding to D3-branes that have played a central role in recent studies of D-brane dynamics and string theory in particular for the AdS/CFT correspondence. The three basic examples of AdS/CFT duality with maximal supersymmetry are provided by taking the Dp -branes to be either M2-branes, D3-branes, or M5-branes. Then the corresponding world volume theories (in 3, 4, or 6 dimensions) have superconformal symmetry. They are conjectured to be dual to M theory or type IIB superstring in space time geometry that is $AdS_4 \times S^7$, $AdS_5 \times S^5$, or $AdS_7 \times S^4$. For example the isometry group of AdS_5 is $SO(2, 4)$ that is also the conformal group in $3 + 1$ dimensions.

It is possible to construct Weyl invariant actions associated to Dp -brane actions for any p by using the same ideas as the ones applied to the case of the p -brane. Here we will show that by introducing a non-dynamical scalar field φ with conformal weight that depends on the dimension we can construct conformal invariant actions for Dp -branes. Using the conformal invariance we promote this scalar field to a dynamical variable. Possible extensions of this ideas can be applied to M-theory five brane action [13], and PST action [14].

This letter is organized as follows, in section 2 we introduce the bosonic Weyl invariant p -brane action, its symmetries and the extended energy momentum tensor. In section 3, the canonical analysis of the p -brane is work out in detail, obtaining the algebra of constraints and identifying the Weyl symmetry generator. In section 4 we construct a Weyl invariant supersymmetric version for any p that is quadratic in $\partial_0 X$. The κ -symmetry associated to this action is now in addition conformal invariant. Finally in section 5 we present the corresponding Weyl invariant action for the Dp -brane, and we promote the auxiliary scalar field to a dynamical variable preserving the conformal symmetry.

2 p -brane Actions

The propose of this section is to explicitly show an action for p -brane systems, that is by construction Weyl invariant in any dimension by adding to the auxiliary metric an auxiliary scalar field with conformal factor that depends on the worldvolume dimension. The transformation properties for this scalar field are fixed at the beginning of the analysis to preserve Weyl invariance and diffeomorphism invariance. The canonical analysis shows a complete consistence of the constraints algebra and symmetry generators as in the case of the non-Weyl invariant p -brane action.

The bosonic Nambu-Goto action for a p -brane is

$$S[X^\mu] = -T \int d^{p+1} \xi \sqrt{-\det g_{ij}}, \quad (1)$$

where T is the p -brane tension, ξ^i , $i = 0, 1, \dots, p$ are the $p + 1$ dimensional world volume coordinates and

$$g_{ij} = g_{\mu\nu} \partial_i X^\mu \partial_j X^\nu, \quad (2)$$

is the worldvolume metric induced by the space-time metric $g_{\mu\nu}$. The non-linear form of this action is inconvenient for canonical analysis and quantization. For that reason it is useful to introduce an auxiliary intrinsic world-volume metric γ_{ij} to write down the following quadratic p -brane action

$$S[X^\mu, \gamma_{ij}] = -\frac{T}{2} \int d^{p+1} \xi \sqrt{-\gamma} \left(\gamma^{ij} g_{ij} - (p-1) \right). \quad (3)$$

Here γ^{ij} is the inverse of γ_{ij} and γ denotes the determinant of γ_{ij} .

This action is invariant under the Weyl transformation $\gamma_{ij} \rightarrow \exp(2\omega) \gamma_{ij}$ only for the case $p = 1$. The role played by this conformal symmetry was crucial to perform the functional integral of the worldsheet metric reducing the problem to a two dimensional Liouville theory. The explicit calculation relies on the fact, special to two dimensions, that it is possible to choice a conformal gauge in such a way that the integration over X^μ yield an integral which depends on the conformal factor through the conformal anomaly. In higher dimensional extended objects, conformal invariance or rather Weyl invariance can be implemented through a higher non-polynomial action. This Weyl invariant extension of the action for the p -brane [5] is

$$S[X^\mu, \gamma_{ij}] = -T \int d^{p+1} \xi \sqrt{-\gamma} \left(\frac{1}{p+1} \gamma^{ij} g_{ij} \right)^{(p+1)/2}. \quad (4)$$

This action has the same or worse drawbacks as the original Nambu-Goto action. It is non-polynomial and difficult to analyze and quantize. The study of this action by using canonical methods is highly non-trivial mainly because the non-linearity inherent in its definition. Indeed, the canonical momenta associated to the space-time coordinates, given by

$$P_\mu = -T \sqrt{-\gamma} \left(\frac{1}{p+1} \gamma^{ij} g_{ij} \right)^{\frac{(p-1)}{2}} \gamma^{0i} \partial_i X_\mu, \quad (5)$$

does not allow us to find the velocities in terms of the momenta for $p > 1$. By this reason we propose the introduction of a new auxiliary scalar field φ in addition to the intrinsic metric γ_{ij} with appropriate Weyl weight to preserve the symmetries of the given action. The action that is now quadratic in ∂X is

$$S[X^\mu, \gamma^{ij}, \varphi] = -\frac{T}{2} \int d^{p+1}\xi \sqrt{-\gamma} \left(\varphi^{1-\frac{2}{p+1}} \gamma^{ij} g_{ij} - \varphi(p-1) \right). \quad (6)$$

The above action is invariant under Weyl symmetry for any p if the scalar field transforms as $\varphi \rightarrow \exp(-\omega(p+1))\varphi$ while the intrinsic worldvolume metric γ_{ij} transforms as usual with the Weyl weight 2ω . Notice that in order to preserve the diffeomorphism invariance of this action the scalar field must also transform under diffeomorphisms as a scalar. This peculiar transformation property of the scalar field φ should be contrasted with the corresponding transformation law of the einbein introduced to remove the square root in the Nambu-Goto action [15].

The action (6) is classically equivalent to the original Nambu-Goto action upon the elimination of the auxiliary fields φ and γ_{ij} by using its own equations of motion. It is also classically equivalent to (4) upon the elimination of the scalar field φ . By fixing the gauge for the Weyl symmetry as $\varphi = 1$ we recover the action for a p -brane given by (3).

The infinitesimal transformations of the fields that leave this action invariant up to a boundary term are

$$\delta X^\mu = \varepsilon^i \partial_i X^\mu, \quad (7)$$

$$\delta \gamma_{ij} = \varepsilon^k \partial_k \gamma_{ij} + \partial_i \varepsilon^k \gamma_{kj} + \partial_j \varepsilon^k \gamma_{ik} + 2\omega \gamma_{ij}, \quad (8)$$

$$\delta \varphi = \varepsilon^i \partial_i \varphi - \omega(p+1)\varphi, \quad (9)$$

where ε^i are $p+1$ infinitesimal arbitrary parameters associated with the diffeomorphism local invariance and ω is an arbitrary parameter responsible of the Weyl local symmetry. Notice the transformation property of the scalar field (9) under local diffeomorphisms.

It is possible to construct the associated energy momentum-tensor for this theory by the prescription

$$T_{ij} \equiv \frac{\varphi^{\frac{2}{p+1}-1}}{\sqrt{-\gamma}} \left(\frac{\delta S}{\delta \gamma^{ij}} + \frac{\varphi}{2} \frac{\delta S}{\delta \varphi} \right) = \frac{1}{p+1} \gamma^{kl} g_{kl} \gamma_{ij} - g_{ij}. \quad (10)$$

This tensor is by definition Weyl invariant and is zero on-shell. Observe that this is not the standard definition of the energy momentum tensor because the intrinsic metric and the auxiliary field transforms under the Weyl symmetry. The solution for γ_{ij} is $\gamma_{ij} = \beta g_{ij}$ with β an arbitrary conformal factor. This tensor has zero trace as a consequence of Weyl invariance and it is the same as the one reported in [16] up to irrelevant factors. It could be interesting to work the trace anomaly for the associated conformal theory in the search of critical dimensions for these conformal invariant theories.

We have also worked out the double dimensional reduction [18] for our conformal action (6) in the case $p = 2$, to recover the conformal invariant string theory. Taking into account that our theory is conformal we can expect some new behavior of this symmetry upon double dimensional reduction. Nevertheless, we found that the conformal theory underlying the compactification is not affected by the original conformal symmetry of the action (6).

3 Canonical analysis of the Weyl invariant p -brane

In this section we develop the canonical formulation of the action (6) in Minkowski space-time and compute the algebra of constraints. To construct the associated canonical analysis for the action (6) we assume that the topology of the world-volume \mathcal{M}^{p+1} is of the form $\Sigma^p \times \mathbb{R}$, where Σ^p is a p dimensional compact manifold. Following the ADM construction we introduce a shift vector N^a ($a = 1, \dots, p$) and a lapse function N . Using these variables the metric of the world-volume γ_{ij} can be written as

$$\gamma_{00} = -N^2 \bar{\gamma} + \bar{\gamma}_{ab} N^a N^b, \quad \gamma_{0a} = \bar{\gamma}_{ab} N^b, \quad \gamma_{ab} = \bar{\gamma}_{ab}, \quad (11)$$

where $\bar{\gamma}_{ab}$ is the intrinsic metric of Σ^p . Substituting (11) in the Lagrangian action (6) we find

$$S = -\frac{T}{2} \int d^{p+1} d\xi \left(\frac{\varphi^{\frac{p-1}{p+1}}}{N} \left[-\dot{X}^2 + 2N^a \dot{X}^\mu \partial_a X_\mu + (N^2 \bar{\gamma} \bar{\gamma}^{ab} - N^a N^b) g_{ab} \right] - (p-1) \varphi N \bar{\gamma} \right). \quad (12)$$

The associated canonical momenta to the configuration variables $(X^\mu, N, N^a, \bar{\gamma}^{ab}, \varphi)$ are

$$P_\mu = T \frac{\varphi^{\frac{p-1}{p+1}}}{N} (\dot{X}_\mu - N^a \partial_a X_\mu), \quad (13)$$

$$\pi \approx 0, \quad \pi_a \approx 0, \quad \pi_{ab} \approx 0, \quad \pi_\varphi \approx 0. \quad (14)$$

The basic Poisson brackets are

$$\begin{aligned} \{X^\mu(\xi), P_\nu(\xi')\} &= \delta^\mu_\nu \delta^p(\xi - \xi'), & \{N(\xi), \pi(\xi')\} &= \delta^p(\xi - \xi'), \\ \{N^a(\xi), \pi_b(\xi')\} &= \delta^a_b \delta^p(\xi - \xi'), & \{\varphi(\xi), \pi_\varphi(\xi')\} &= \delta^p(\xi - \xi'), \\ \{\bar{\gamma}^{ab}(\xi), \pi_{cd}(\xi')\} &= \frac{1}{2} (\delta^a_c \delta^b_d + \delta^b_c \delta^a_d) \delta^p(\xi - \xi'). \end{aligned} \quad (15)$$

From the definition of the momenta we obtain $\frac{(p+1)(p+2)}{2} + 1$ primary constraints. The total Hamiltonian associated to (12) is

$$\begin{aligned} H_T &= \int d^p \xi \left(\frac{N}{2} \left(\frac{1}{T} \varphi^{\frac{1-p}{p+1}} P_\mu P^\mu + T \varphi^{\frac{p-1}{p+1}} \bar{\gamma} \bar{\gamma}^{ab} g_{ab} - T \varphi \bar{\gamma} (p-1) \right) + N^a P_\mu \partial_a X^\mu \right. \\ &\quad \left. + \lambda_\varphi \pi_\varphi + \lambda \pi + \lambda^a \pi_a + \lambda^{ab} \pi_{ab} \right), \end{aligned} \quad (16)$$

here the λ 's are the Lagrangian multipliers associated to the primary constraints. By using the Dirac method the evolution in time of these constraints generate the following $(p+1)(p+2)/2$ secondary constraints

$$\begin{aligned} \mathcal{H} &= \frac{1}{2} \left(\frac{1}{T} P_\mu P^\mu + T \varphi^{2-\frac{4}{p+1}} \bar{\gamma} \bar{\gamma}^{ab} g_{ab} - T \varphi^{2-\frac{2}{p+1}} \bar{\gamma} (p-1) \right) \approx 0, \\ \mathcal{H}_a &= P_\mu \partial_a X^\mu \approx 0, \\ \Omega_{ab} &= T (g_{ab} - \varphi^{\frac{2}{p+1}} \bar{\gamma}_{ab}) \approx 0. \end{aligned} \quad (17)$$

The evolution in time of these secondary constraints does not produce new constraints. To split the constraints according to its first or second class character we observe that the constraints Ω_{ab} and π_{ab} are second class. This are $p(p+1)$ constraints. Furthermore, from the range of the matrix defined by the Poisson brackets between all the constraints, we conclude that there are no more second class constraints. By a redefinition of the constraints \mathcal{H} , \mathcal{H}_a and π_φ on the constraint surface, the algebra of the complete set of $2p+3$ first class constraints can be closed up to quadratic pieces in second class constraints. To that end we propose the following complete set of constraints

First class:

$$\pi \approx 0, \quad \pi_a \approx 0, \quad (18)$$

$$\mathcal{T}_\varphi \equiv \pi_\varphi + \frac{2}{p+1} \varphi^{-\frac{2}{p+1}-1} \pi^{ab} g_{ab} \approx 0, \quad (19)$$

$$\mathcal{T} \equiv \mathcal{H} + \frac{2}{T} P_\mu \partial_a (\varphi^{-\frac{2}{p+1}} \pi^{ab} \partial_b X^\mu) \approx 0, \quad (20)$$

$$\mathcal{T}_a \equiv \mathcal{H}_a + 2 \partial_a X^\mu \partial_b (\varphi^{-\frac{2}{p+1}} \pi^{bc} \partial_c X_\mu) \approx 0. \quad (21)$$

Second class:

$$\pi_{ab} \approx 0, \quad \Omega_{ab} \approx 0. \quad (22)$$

The Poisson bracket between the second class constraints is

$$\{\Omega_{ab}(\xi), \pi_{cd}(\xi')\} = \frac{1}{2} (\bar{\gamma}_{ac} \bar{\gamma}_{bd} + \bar{\gamma}_{ad} \bar{\gamma}_{bc}) \varphi^{\frac{2}{p+1}} \delta^p(\xi - \xi') \quad (23)$$

To compute the algebra *on* the second class constraint surface we introduce the Dirac bracket,

$$\begin{aligned} \{F, G\}^* &= \{F, G\} + \frac{1}{T} \left(\int d^p \xi \{F, \Omega_{ab}(\xi)\} \varphi^{\frac{-2}{p+1}} \bar{\gamma}^{ac} \bar{\gamma}^{bd}(\xi) \{\pi_{cd}(\xi), G\} \right. \\ &\quad \left. - \int d^p \xi \{F, \pi_{ab}(\xi)\} \varphi^{\frac{-2}{p+1}} \bar{\gamma}^{ac} \bar{\gamma}^{bd}(\xi) \{\Omega_{cd}(\xi), G\} \right). \end{aligned} \quad (24)$$

The relevant Dirac brackets between the canonical variables are

$$\begin{aligned} \{X^\mu(\xi), P_\nu(\xi')\}^* &= \delta^\mu_\nu \delta^p(\xi - \xi'), & \{N(\xi), \pi(\xi')\}^* &= \delta^p(\xi - \xi'), \\ \{N^a(\xi), \pi_b(\xi')\}^* &= \delta^a_b \delta^p(\xi - \xi'), & \{\varphi(\xi), \pi_\varphi(\xi')\}^* &= \delta^p(\xi - \xi'), \\ \{\bar{\gamma}^{ab}(\xi), \pi_{cd}(\xi')\}^* &= 0, \\ \{\bar{\gamma}^{ab}(\xi), P_\mu(\xi')\}^* &= \varphi^{-\frac{2}{p+1}} \bar{\gamma}^{ac} \bar{\gamma}^{bd} [\partial_a X_\mu \partial_c \delta^p(\xi - \xi') + \partial_c X_\mu \partial_a \delta^p(\xi - \xi')], \\ \{\bar{\gamma}^{ab}(\xi), \pi_\varphi(\xi')\}^* &= \frac{2}{\varphi(p+1)} \bar{\gamma}^{ab} \delta^p(\xi - \xi'). \end{aligned} \quad (25)$$

Using these Dirac brackets the full Dirac algebra of the densitized constraints is

$$\begin{aligned} \{F[f], \Omega_{ab}[g]\}^* &= \{F[f], \pi_{ab}[g]\}^* = 0, \\ \{\mathcal{T}_\varphi[f], \mathcal{T}[g]\}^* &= \{\mathcal{T}_\varphi[f], \mathcal{T}_a[g]\}^* = 0, \\ \{\mathcal{T}[f], \mathcal{T}[g]\}^* &= \mathcal{T}_a[(f \partial_b g - g \partial_b f) \varphi^{(2-\frac{4}{p+1})} \bar{\gamma} \bar{\gamma}^{ab}], \\ \{\mathcal{T}[f], \mathcal{T}_a[g]\}^* &= \mathcal{T}[f \partial_a g - g \partial_a f], \\ \{\mathcal{T}_a[f], \mathcal{T}_b[g]\}^* &= \mathcal{T}_b[f \partial_a g] - \mathcal{T}_a[g \partial_b f]. \end{aligned} \quad (26)$$

where F is any function of the canonical variables.

From this analysis we can conclude that the standard diffeomorphism algebra for the p -brane is reproduced but with different structure functions that are now modified to preserve the Weyl symmetry. Compared to the standard non-conformal p -brane action (3) here we have a new symmetry generator $-\omega(p+1)\varphi\mathcal{T}_\varphi$ for the Weyl local transformation through the corresponding Dirac brackets. From constraint analysis we obtain the number of physical degrees of freedom in the following way. We have $D+1+(p+1)(p+2)/2$ configuration variables, $2p+3$ first class constraints and $p(p+1)$ second class constraints. Then the number of physical degrees of freedom per space-time point is, $D-(p+1)$.

4 Conformal Actions for Super p -Brane

In previous sections we show how to build Weyl invariant actions for bosonic p . In this section we will extend the construction of the Weyl invariant p -brane action (6) to the supersymmetric Green-Schwarz type actions in curved space-time.

Following the construction of the super p -brane in [17] we add to the action (6) a Wess-Zumino term ¹

$$S[X^\mu, \gamma_{ij}, \varphi] = -\frac{T}{2} \int d^{p+1}\xi \left\{ \sqrt{-\gamma} \left(\varphi^{1-\frac{2}{p+1}} \gamma^{ij} \partial_i X^\mu \partial_j X^\nu G_{\mu\nu} - \varphi(p-1) \right) + \frac{2}{(p+1)!} \varepsilon^{i_1 i_2 \dots i_{p+1}} \partial_{i_1} X^{\mu_1} \partial_{i_2} X^{\mu_2} \dots \partial_{i_{p+1}} X^{\mu_{p+1}} B_{\mu_{p+1} \dots \mu_2 \mu_1} \right\}, \quad (27)$$

where $G_{\mu\nu}$ is the background metric and B is an antisymmetric tensor. This action (27) preserves the Weyl symmetry under transformations of the intrinsic metric and auxiliary field proposed in section 2. To obtain the super p -brane action, we introduce the coordinates Z^M of the curved super-space-time

$$Z^M = (X^\mu, \theta^\alpha), \quad (28)$$

and the supervielbein $E_M^A(Z)$, where $M = \mu, \alpha$ are super-space-time indices and $A = a, \alpha$ are indices in the associated tangent space. Defining the supervielbein pull-back

$$E_i^A = \partial_i Z^M E_M^A, \quad (29)$$

the action in the superspace can be written as

$$S[Z^M, \gamma_{ij}, \varphi] = -\frac{T}{2} \int d^{p+1}\xi \left\{ \sqrt{-\gamma} \left(\varphi^{1-\frac{2}{p+1}} \gamma^{ij} E_i^a E_j^b \eta_{ab} - \varphi(p-1) \right) + \frac{2}{(p+1)!} \varepsilon^{i_1 \dots i_{p+1}} E_{i_1}^{A_1} \dots E_{i_{p+1}}^{A_{p+1}} B_{A_{p+1} \dots A_1} \right\}. \quad (30)$$

This action is invariant under the super world volume diffeomorphisms and Weyl transformations

$$\begin{aligned} \delta Z^M &= \eta^i \partial_i Z^M, & \delta \varphi &= \eta^i \partial_i \varphi - \omega(p+1)\varphi, \\ \delta \gamma_{ij} &= \eta^k \partial_k \gamma_{ij} + \partial_i \eta^k \gamma_{jk} + \partial_j \eta^k \gamma_{ik} + 2\omega \gamma_{ij}, \end{aligned} \quad (31)$$

¹Here p and the space-time dimension are restricted by the brane scan [19].

where η^i are the infinitesimal parameters associated with the diffeomorphism transformation. Furthermore, it is invariant under supersymmetric local κ transformations, given by

$$\begin{aligned}
\delta E^a &= 0, \\
\delta E^\alpha &= (1 + \Gamma)^\alpha_\beta \kappa^\beta, \\
\delta(\sqrt{-\gamma} \varphi^{1-\frac{2}{p+1}} \gamma^{ij}) &= -\frac{4i\zeta}{p!} (1 + \Gamma)^\alpha_\beta \kappa^\beta (\Gamma_{a_1 \dots a_p})_{\gamma\alpha} E_n^\gamma \varphi^{-\frac{2}{p+1}} \gamma^{ni} \varepsilon^{ji_1 \dots i_p} E_{i_1}^{a_1} \dots E_{i_p}^{a_p} \\
&\quad + \frac{2}{(p+1)! \varphi \sqrt{-\gamma}} (-2i \varphi^{-\frac{2}{p+1}} \gamma^{lm} E_l^\alpha E_m^a (\Gamma_a)_{\alpha\beta} + (p+1) \Lambda_\beta) \kappa^\beta \\
&\quad \varepsilon^{i_1 \dots i_p i} \varepsilon^{jj_p \dots j_1} (E_{i_1}^{a_1} E_{j_1 a_1} \dots E_{i_p}^{a_p} E_{j_p a_p} + \\
&\quad \varphi^{\frac{2}{p+1}} E_{i_1}^{a_1} E_{j_1 a_1} \dots E_{i_{p-1}}^{a_{p-1}} E_{j_{p-1} a_{p-1}} \gamma_{i_p j_p} + \dots + \varphi^{\frac{2p}{p+1}} \gamma_{i_1 j_1} \dots \gamma_{i_p j_p}) \\
&\quad + 2\sqrt{-\gamma} \varphi^{1-\frac{2}{p+1}} \gamma^{ij} \delta E^\beta \Lambda_\beta,
\end{aligned} \tag{32}$$

where $\kappa^\beta(\xi)$ is a spinor in the space-time and a scalar in the worldvolume. Here we have used the notation

$$\delta E^A \equiv \delta Z^M E_M^A, \tag{33}$$

where $(\Gamma)^\alpha_\beta$ and Λ_β are defined in terms of the torsion of the super space

$$(\Gamma)^\alpha_\beta = \frac{\zeta}{(p+1)! \varphi \sqrt{-g}} \varepsilon^{i_1 \dots i_{p+1}} E_{i_1}^{a_1} \dots E_{i_{p+1}}^{a_{p+1}} (\Gamma_{a_1 \dots a_{p+1}})^\alpha_\beta, \tag{34}$$

$$\begin{aligned}
T_{\alpha\beta}^a &= -2i(\Gamma^a)_{\alpha\beta}, & \eta_{c(a} T_{b)\alpha}^c &= \eta_{ab} \Lambda_\alpha, \\
H_{\alpha a_{p+1} \dots a_1} &= (p+1) \Lambda_\beta (\Gamma_{a_1 \dots a_{p+1}})^\beta_\alpha, \\
H_{\alpha\beta a_p \dots a_1} &= 2i\zeta(-1)^{p+1} (\Gamma_{a_1 \dots a_p})_{\beta\alpha}, \\
H_{\alpha\beta\gamma A_1 \dots A_{p-1}} &= 0,
\end{aligned} \tag{35}$$

with $\zeta = (-1)^{(p+1)(p-2)/4}$. Notice that the form of this κ -transformation is very similar to the previously one found in [17], with the remark that in our case the κ -transformation (32) preserves the Weyl invariance. The action (30) is quadratic in $\partial_0 X$ allowing also a detailed canonical analysis [20].

5 Weyl invariant action for Dp -brane

The Born-Infeld-type action for Dp -brane is

$$S = -T \int d^{p+1} \xi \exp(-\phi) \sqrt{-\det(g_{ij} + \mathcal{F}_{ij})}, \tag{36}$$

where

$$\mathcal{F}_{ij} \equiv F_{ij} - B_{ij}, \tag{37}$$

and ϕ , g_{ij} and B_{ij} are the pullbacks to the world-volume of the background dilaton, metric and NS antisymmetric two-form fields and $F_{ij} = \partial_i A_j - \partial_j A_i$ is the field strength of the

$U(1)$ world-volume gauge field A_i . The action (36) was rewritten in a more geometric way in [12, 16] introducing an intrinsic metric γ_{ij} , obtaining the classically equivalent form

$$S = -\frac{T}{4} \int d^{p+1} \xi \exp(-\phi) (-g)^{1/4} (-\gamma)^{1/4} \left[\gamma^{ij} (g_{ij} - g^{kl} \mathcal{F}_{ik} \mathcal{F}_{lj}) - (p-3) \right]. \quad (38)$$

This action is the analog of the Polyakov action for the string and has the characteristic that is invariant under Weyl transformations

$$\gamma_{ij} \rightarrow \exp(2\omega) \gamma_{ij}, \quad (39)$$

only for $p = 3$. This property can be extended to any dimension using an alternative action to (38). The form of the action that is Weyl invariant for any p is

$$S = -T \int d^{p+1} \xi \exp(-\phi) (-g)^{1/4} (-\gamma)^{1/4} \left[\frac{1}{p+1} \gamma^{ij} (g_{ij} - g^{kl} \mathcal{F}_{ik} \mathcal{F}_{lj}) \right]^{\frac{p+1}{4}}. \quad (40)$$

Using the equation of motion for γ_{ij} , the action (40) reduces to the Born-Infeld action (36). Also, we can see that for $p = 3$ the action (40) is equal to (38). However, for any dimensions, we have again the problem that this action is highly non-linear and then difficult to analyze and quantize. To solve this difficulty we introduce, in a similar way to the case of the p -brane, an auxiliary field φ that eliminates the power of $\frac{p+1}{4}$ in the action. The resulting expression is

$$S = -\frac{T}{4} \int d^{p+1} \xi \exp(-\phi) (-g)^{1/4} (-\gamma)^{1/4} \left[\gamma^{ij} (g_{ij} - g^{kl} \mathcal{F}_{ik} \mathcal{F}_{lj}) \varphi^{\frac{p-3}{p+1}} - (p-3) \varphi \right]. \quad (41)$$

The above action is invariant under the Weyl transformation (39) iff the auxiliary field φ transforms as

$$\varphi \rightarrow \exp\left(-\frac{\omega}{2}(p+1)\right) \varphi, \quad (42)$$

and in this way compensates the transformations of the intrinsic metric γ_{ij} . Furthermore, from the equation of motion for φ we get

$$\varphi = \left(\frac{1}{p+1} \gamma^{ij} (g_{ij} - g^{kl} \mathcal{F}_{ik} \mathcal{F}_{lj}) \right)^{\frac{p+1}{4}}. \quad (43)$$

Using this expression for φ in (41) we recover (40). This shows that both actions are classically equivalent and also Weyl invariant for any p . Nevertheless, the action (41) is quadratic in the field strength \mathcal{F}_{ij} for any dimension whereas (40) is quadratic only for $p = 3$.

For the action (40) we define a Weyl invariant energy-momentum tensor T^{ij} using the expression

$$\begin{aligned} T^{ij} &= -\frac{1}{T} (-\gamma)^{\frac{1}{p+1}} \frac{\delta S}{\delta \gamma_{ij}} \\ &= \exp(-\phi) (-g)^{1/4} (-\gamma)^{\frac{p+5}{4(p+1)}} \left(\frac{\gamma \cdot G}{p+1} \right)^{\frac{p-3}{4}} \left[\gamma^{ij} \left(\frac{\gamma \cdot G}{p+1} \right) - \gamma^{ki} \gamma^{lj} G_{kl} \right], \end{aligned} \quad (44)$$

where $G_{ij} = g_{ij} - g^{kl}\mathcal{F}_{ik}\mathcal{F}_{lj}$ and $\gamma \cdot G = \gamma^{ij}G_{ij}$. This tensor is traceless $\gamma_{ij}T^{ij} = 0$ for any dimension as a result of the Weyl invariance. For the action (41) we can introduce an equivalent energy-momentum tensor that is also traceless and is given by

$$\begin{aligned} T^{ij} &= -\frac{1}{T}(-\gamma)^{\frac{1}{p+1}} \left(\frac{\delta S}{\delta \gamma_{ij}} - \frac{\varphi}{4} \gamma^{ij} \frac{\delta S}{\delta \varphi} \right) \\ &= \exp(-\phi)(-g)^{1/4}(-\gamma)^{\frac{p+5}{4(p+1)}} \varphi^{\frac{p-3}{p+1}} \left[\gamma^{ij} \left(\frac{\gamma \cdot G}{p+1} \right) - \gamma^{ki} \gamma^{lj} G_{kl} \right]. \end{aligned} \quad (45)$$

By using the equation of motion (43), the energy-momentum tensor (45) is exactly the same as (44).

From this new form of the Weyl invariant action for Dp-branes (41), we can construct new conformal couplings of the scalar field with the gauge fields on the brane. The transformation law for the scalar field under diffeomorphisms suggest that it can be promoted to a dynamical field playing the role analogous to a space-time coordinate. Considering that the transformation (42) is equivalent to a U(1) gauge transformation with imaginary parameter, a natural way to minimally couple the scalar field φ to the gauge fields A_i is by the introduction of the Weyl covariant derivative [21],

$$\mathcal{D}_i = \partial_i + A_i, \quad (46)$$

and requiring that the vector potential A_i transform under the Weyl symmetry as a U(1) gauge connection, *i.e.*,

$$A_i \rightarrow A_i + \frac{p+1}{2} \partial_i \omega. \quad (47)$$

As a consequence of this property the covariant derivative turns out to be

$$\mathcal{D}_i \varphi \rightarrow \exp \left(\frac{\omega}{2} (p+1) \right) \mathcal{D}_i \varphi. \quad (48)$$

A Weyl invariant action with the scalar field φ promoted to a dynamical variable is

$$S = -\frac{T}{4} \int d^{p+1} \xi \exp(-\phi)(-g)^{1/4}(-\gamma)^{1/4} \left[\gamma^{ij} G_{ij} \varphi^{\frac{p-3}{p+1}} - (p-3) \varphi \right]. \quad (49)$$

where the new induced metric is

$$g_{ij} = g_{\mu\nu} \partial_i X^\mu \partial_j X^\nu + \varphi^{-2} \mathcal{D}_i \varphi \mathcal{D}_j \varphi. \quad (50)$$

Notice that in addition to the standard space-time induced metric, the scalar and the gauge fields modify the induced metric on the brane. In this sense these fields induce curvature on the brane.

By the elimination of the auxiliary metric γ_{ij} we recover a Born-Infeld type action (36) with the induced metric given by (50). Notice that this conformal coupling is different from the one proposed in [21], where the induced metric g_{ij} does not depends on the scalar field and transform under the conformal symmetry. In this model it is also possible to incorporate world-volume dynamical gravity in a Weyl invariant way by adding a fourth derivative Weyl term or by using the Einstein term with the compensator φ [21, 22].

In this letter we have constructed Weyl invariant versions of the p -brane and Dp -brane bosonic actions and the space-time supersymmetric extension for the p -brane case by using an auxiliary scalar field. One important feature that emerges from the p -brane action is that it is quadratic in the associated velocities \dot{X} . As a consequence of this fact the canonical analysis was work out in detail and it may be possible to obtain some relevant quantum properties as the potential existence of a conformal anomaly associated to the stress tensor (10), and the analysis of the potential appearance of critical dimensions.

In our discussion of the space-time supersymmetric case we observe that the κ symmetry constructed in (32) is now conformal invariant. For the spinning membrane case the construction of Weyl invariant actions using auxiliary scalar fields has been proposed recently in [23].

We have not investigated about if the auxiliary scalar field can be used to parameterize the conformal diffeomorphisms that are a subclass of the remaining symmetries after a covariant gauge has been imposed.

An interesting property of the conformal Dp -brane action is that it is now possible to use the conformal symmetry as a guiding principle to construct couplings of the auxiliary field with the dynamical fields of the theory. The conformal symmetry induces couplings to the gauge fields and deform the induced metric on the brane.

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